



Pinfold Primary

'Learning is our passion'

'Small but mighty!'

Mathematics Progression through
Calculations Policy 2020
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Chair of Governors: Nicola Jackson

For all calculations:

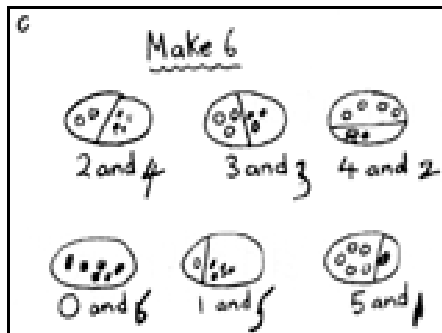
- These standards are age-related expectations and therefore we expect the majority of children to achieve them.
- New learning is likely to be taught to groups rather than the whole class to acknowledge the different learning stages of the children.

Progression through calculations for addition

- Children should understand that addition is commutative and therefore calculations can be rearranged, e.g. $4 + 13 = 17$ is the same as $13 + 4 = 17$.
- Ensure that children understand the = sign means is the same as, not makes, and that children see calculations where the equals sign is in a different position, e.g. $3 + 2 = 5$ and $5 = 3 + 2$.
- Children should be encouraged to approximate before calculating and check whether their answer is reasonable.

YR

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc.



Children **who are ready** may record this as:

$$6 = 2 + 4 \quad 6 = 3 + 3 \quad 6 = 4 + 2 \quad 6 = 0 + 6 \quad 6 = 1 + 5 \quad 6 = 5 + 1$$

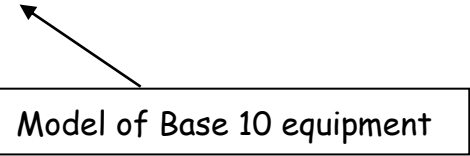
Y1

Children will initially use practical equipment to combine groups of objects to find the total. They will move on to the use of number tracks and Base 10 equipment to support their developing understanding of addition. If possible, use two different colours of base 10 equipment so that the initial amounts can still be seen.

$$11 + 5 =$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

$$11 + 5 = 16$$

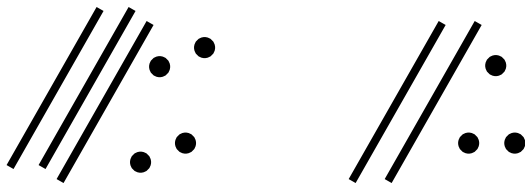


Y2

Children will continue to use the Base 10 equipment to support their calculations. They will record the calculations using their own drawings of the Base 10 equipment (as lines for the 10 rods and dots for the unit blocks).

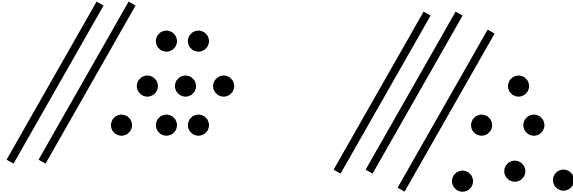
They would add the units first and then count on the tens.

e.g. $34 + 23 =$



$$\begin{array}{r}
 34 + 23 = \quad 4 + 3 = 7 \\
 30 + 20 = \underline{50} \\
 \quad \quad \quad 57
 \end{array}$$

e.g. $28 + 36 =$



$$\begin{array}{r}
 28 + 36 = \quad 8 + 6 = 14 \\
 20 + 30 = \underline{50} \\
 \quad \quad \quad 64
 \end{array}$$

When the units total more than 10, children should be encouraged to exchange 10 ones for 1 ten. This is the start of children understanding 'carrying' in vertical addition.

Y3

Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use this to support with the transition into a vertical method.

Children should add the **units** first as preparation for the compact method.

TU	
67	
+ 24	

11 (7 + 4)	

80 (60 + 20)	

91	

HTU	
267	
+ 85	

12 (7 + 5)	

140 (60 + 80)	

200	

352	

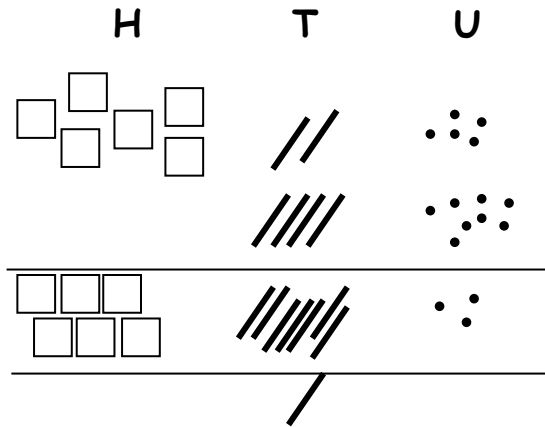
The Base 10 equipment should be used alongside to model the transition to the vertical method. The children may write the information at the side as a support at first then remove this when they are more confident.

Y4

Based on their experiences in Y3, children will then begin to carry below the line.

The best way to model this would be using Base 10 equipment to show how units would transfer to tens.

$$\begin{array}{r} \text{HTU} \\ 625 \\ + 48 \\ \hline 673 \\ \underline{1} \end{array}$$



Teacher model

$$\begin{array}{r} 783 \\ + 42 \\ \hline 825 \\ \underline{1} \end{array}$$

$$\begin{array}{r} 367 \\ + 85 \\ \hline 452 \\ \underline{11} \end{array}$$

$$\begin{array}{r} 321 \\ + 7 \\ \hline + 48 \\ \hline 376 \\ \underline{1} \end{array}$$

$$\begin{array}{r} \text{£}3.48 \\ + \text{£}0.78 \\ \hline \text{£}4.26 \\ \underline{11} \end{array}$$

Using similar methods, children will:

- add several numbers with different numbers of digits;
- begin to add two or more three-digit sums of money
- know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.

Y5

Children should extend the carrying method to numbers with at least four digits.

$$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ \underline{11} \end{array}$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ \underline{111} \end{array}$$

$$\begin{array}{r} 3121 \\ + 37 \\ \hline + 148 \\ \hline 3306 \\ \underline{11} \end{array}$$

$$\begin{array}{r} 3.20 \\ + 2.88 \\ \hline 6.08 \\ \underline{1} \end{array}$$

Using similar methods, children will:

- add several numbers with different numbers of digits;
- begin to add two or more decimal fractions with up to three digits and the same number of decimal places;
- know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m + 280 cm.

Y6

Children should extend the carrying method to number with any number of digits.

$$\begin{array}{r} 6584 \\ + 5848 \\ \hline 12432 \\ \text{1 1 1} \end{array} \quad \begin{array}{r} 42 \\ 6432 \\ 786 \\ 3 \\ + 4681 \\ \hline 11944 \\ \text{1 2 1} \end{array} \quad \begin{array}{r} 401.20 \\ + 26.85 \\ + 0.71 \\ \hline 428.76 \\ 1 \end{array}$$

Using similar methods, children will

- add several numbers with different numbers of digits;
- begin to add two or more decimal fractions with up to four digits and either one or two decimal places;
- know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. $401.2 + 26.85 + 0.71$.

+ - + - + - + - + - + - +

Children should not be made to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

- Children should understand that subtraction is the removing or taking away one quantity from another (not necessarily the smaller number from the larger one) or finding the difference between two separate quantities.
- Children should understand that, unlike addition, subtraction is **not** commutative.
- Ensure that children understand the = sign means is the same as, not makes, and that children see calculations where the equals sign is in a different position, e.g. $9 - 5 = 4$ and $4 = 9 - 5$.
- Children should be encouraged to approximate before calculating and check whether their answer is reasonable.

YR

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc.

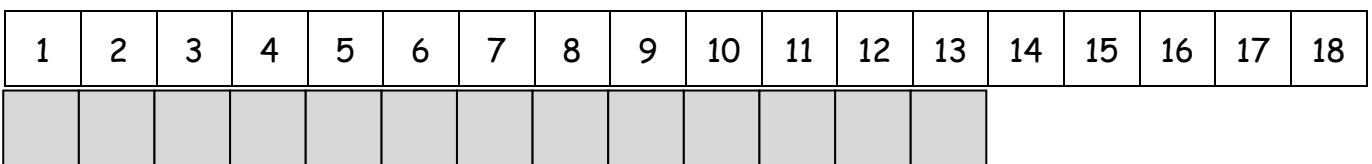


Children who are ready may record this as $8 - 5 = 3$

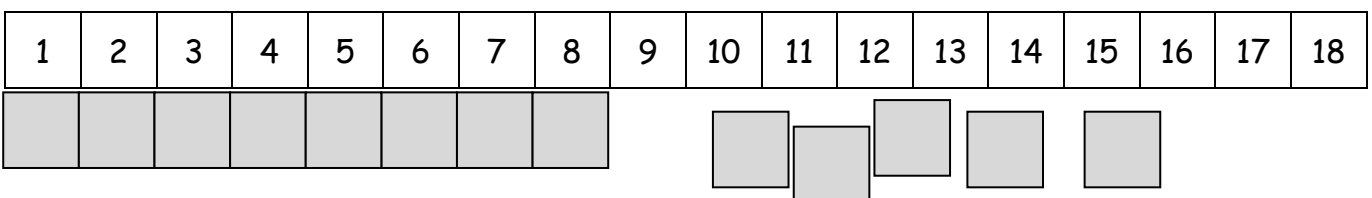
Y1

Children will use practical equipment for subtraction by taking away (counting back).

$13 - 5 =$



Count out 13 cubes along the number track followed by removal of 5 cubes to give answer:



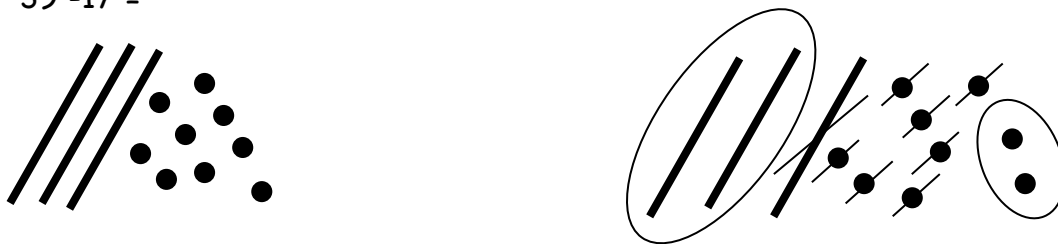
$13 - 5 = 8$

It is important that children keep track of how many have been removed.

Y2

Children will move on to using the Base 10 equipment to support their calculations. They need to understand that the number being subtracted does not appear as an amount on its own, but rather as part of the larger amount.

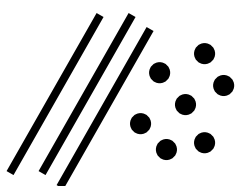
e.g. $39 - 17 =$



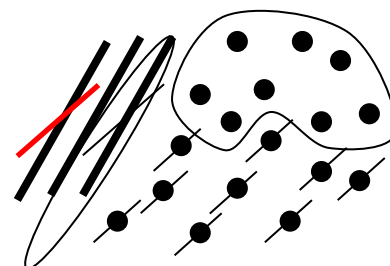
Children would count out 39 using the Base 10 equipment (3 tens and 9 units) and would remove 7 units and then one ten, counting up the answer of 2 tens and 2 units to give 22.

When exchange is required:

$37 - 19 =$



Children can see that they cannot subtract 9 units from 7 units so they need to exchange a ten for ten units. This will become:




Children would count out how many tens and units are left to give the answer (18).

At the end of Y2, children will record this by drawing representations of the Base 10 material and crossing out those pieces that they are subtracting. If children are representing exchange, they should cross out a 10 rod line and replace with 10 unit dots.

Y3

Children should begin the method of expanded decomposition with, initially, TU - TU calculations. This process should be demonstrated using arrow cards to show the partitioning and Base 10 materials to show the decomposition of the number.



| | | | | | |
|-------------|---|-----------|---|------|--|
| 89 | = | 80 | → | 9 | The calculation should be read as subtract 7 from 9 or 9 subtract 7. |
| <u>- 57</u> | | <u>50</u> | → | 7 | |
| | | 30 | → | 2 | |
| | | | | = 32 | |

Children should use the Base 10 materials to represent the first number and remove the units and tens as appropriate (as with the more informal method in Y2).

Initially, the children will be taught using examples that do not need the children to exchange. Emphasise that the bottom number is being subtracted from the top number rather than the smaller number from the bigger.

From this the children will begin to solve problems which involve exchange:

$$\begin{array}{r} 71 \\ - 46 \\ \hline \end{array}$$

$$\begin{array}{r} 70 \rightarrow 1 \\ - 40 \rightarrow 6 \\ \hline \end{array}$$

The calculation should be read as subtract 6 from 1 or 1 subtract 6.

Children can see that they cannot subtract 6 units from the 1 unit so they need to exchange a ten for ten units. This will become:

$$\begin{array}{r} \cancel{70} \rightarrow 1 \\ - 40 \rightarrow 6 \\ \hline 20 \rightarrow 5 = 25 \end{array}$$

(Stop drawing the base 10 when the children are ready.)

Y4

$$\begin{array}{r} \cancel{600} \rightarrow \cancel{50} \rightarrow 14 \\ - \phantom{\cancel{50}} \\ \\ \hline 600 \rightarrow 60 \rightarrow 8 = 668 \end{array}$$

When children are ready, this leads on to the compact method of decomposition:

$$\begin{array}{r} 614 \\ \cancel{7} \cancel{5} 4 \\ - 86 \\ \hline 668 \end{array}$$

Children should:

- be able to subtract numbers with different numbers of digits.

Y5

If children have not reached the stage of compact method of decomposition then they will continue at this point with the expanded method.

$$\begin{array}{r} 614 \\ \cancel{1} \cancel{7} \cancel{5} 4 \\ - 286 \\ \hline 1468 \end{array} \qquad \begin{array}{r} 2 1 \\ \cancel{2} . \cancel{4} 2 \\ - . 176 \\ \hline 1.66 \end{array}$$

Children should:

- be able to subtract numbers with different numbers of digits;
- begin to subtract two decimal fractions with up to three digits and the same number of decimal places;

Y6

$$\begin{array}{r} \\ 1 \cancel{6} 4 6 7 \\ - \underline{2684} \\ 13783 \end{array}$$

Children should:

- be able to subtract numbers with different numbers of digits;
- be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;

+ - + - + - + - + - + - +

Children ***should not be made*** to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Progression Towards a Written Method for Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

- repeated addition

They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:

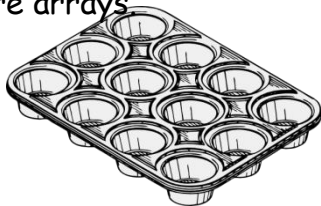
- the inverse of division
- commutative i.e. 5×3 is the same as 3×5
- associative i.e. $2 \times 3 \times 5$ is the same as $2 \times (3 \times 5)$

YR

Early Learning Goal:
Children solve problems, including doubling.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays



They may develop ways of recording calculations using pictures, etc.



A child's jotting showing the fingers on each hand as a double.



A child's jotting showing double three as three cookies on each plate.

Y1

End of Year Objective:

Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

In year one, children will continue to solve multiplication problems using practical equipment and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'

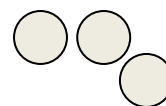
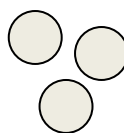
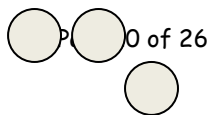
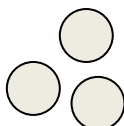
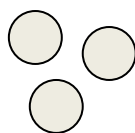
Y2

End of Year Objective:

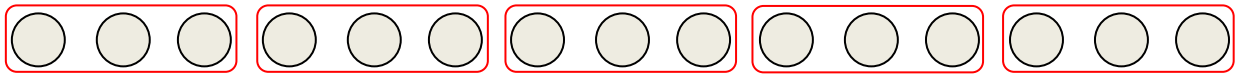
Calculate mathematical statements for multiplication (using repeated addition) and write them using the multiplication (x) and equals (=) signs.

Children should understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes. e.g.

5 x 3 can be shown as five groups of three with counters, either grouped in a random pattern, as below:

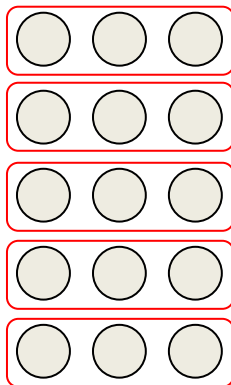


or in a more ordered pattern, with the groups of three indicated by the border outline:

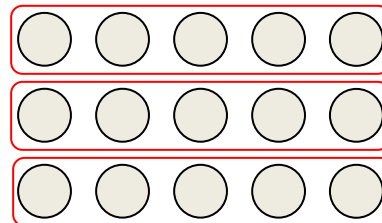


Children should then develop this knowledge to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future). Again, children should be encouraged to use practical apparatus and jottings to support their understanding, e.g.

$5 \times 3^*$ can be represented as an array in two forms (as it has commutativity):



$$3 + 3 + 3 + 3 + 3 = 15$$



$$5 + 5 + 5 = 15$$

*For mathematical accuracy 5×3 is represented by the second example above, rather than the first as it is five, three times. However, because we use terms such as 'groups of' or 'lots of', children are more familiar with the initial notation. Once children understand the commutative order of multiplication the order is irrelevant).

Y3

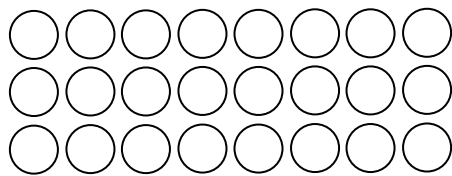
End of Year Objective:

Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.

Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$$3 \times 8$$

They may show this using practical equipment:



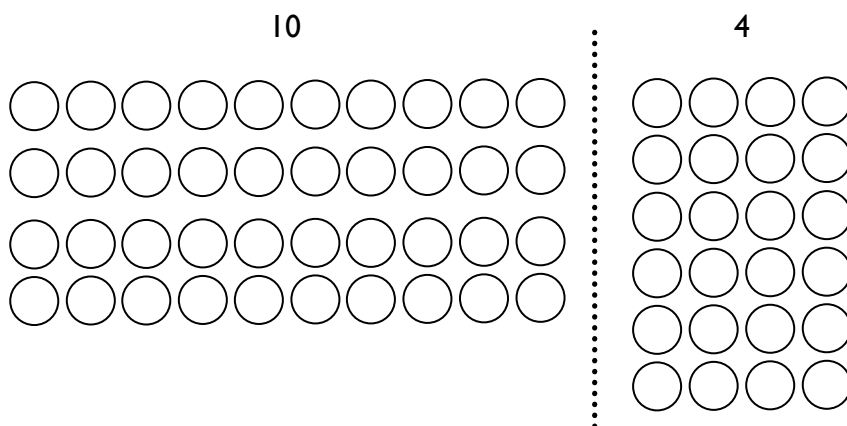
$$3 \times 8 = 8 + 8 + 8 = 24$$

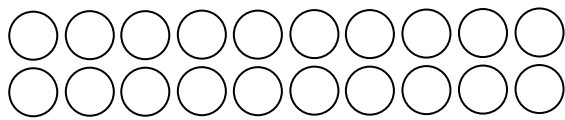
or by jottings using squared paper:

| | | | | | | | | | |
|--|---|---|---|---|---|---|---|---|--|
| | x | x | x | x | x | x | x | x | |
| | x | x | x | x | x | x | x | x | |
| | x | x | x | x | x | x | x | x | |
| | | | | | | | | | |

$$3 \times 8 = 8 + 8 + 8 = 24$$

As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and units/ones to help them. For example, when calculating 14×6 , children should set out the array, then partition the array so that one array has ten columns and the other four.





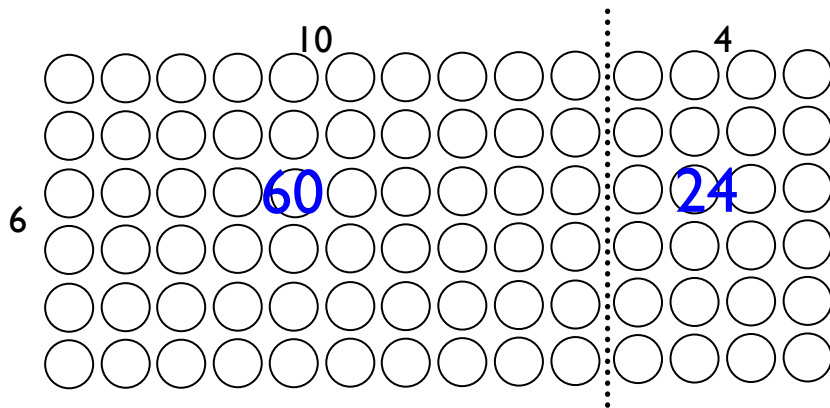
Partitioning in this way, allows children to identify that the first array shows 10×6 and the second array shows 4×6 . These can then be added to calculate the answer:

$$(6 \times 10) + (6 \times 4)$$

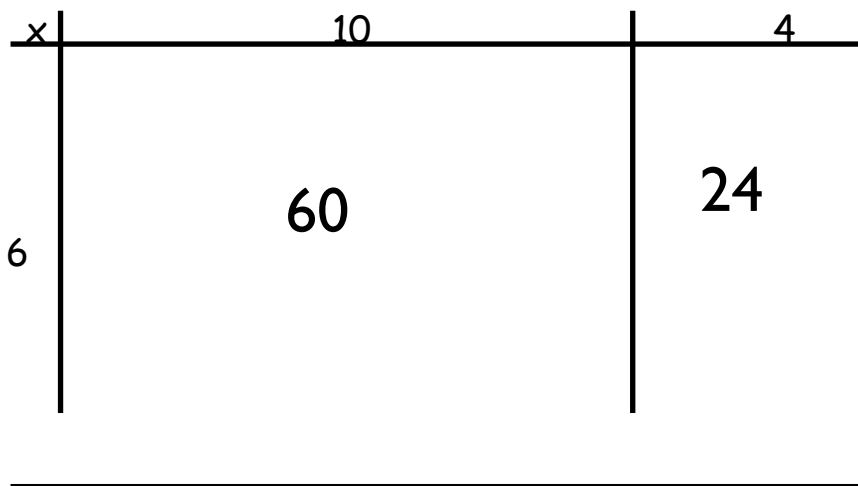
$$= 60 + 24 = 84$$

NB There is no requirement for children to record in this way, but it could be used as a jotting to support development if needed.

This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line.



By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.



It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

To support the grid method, children should develop their understanding of place value and facts that are linked to their knowledge of tables. For example, in the calculation above, children should use their knowledge that $7 \times 8 = 56$ to know that $70 \times 8 = 560$.

$$13 \times 8$$

| | | |
|---|----|----|
| x | 10 | 3 |
| 8 | 80 | 24 |

$$\begin{array}{r} 80 \\ + 24 \\ \hline 104 \end{array}$$

When children are ready, they can then progress to using this method with other two-digit numbers.

$$37 \times 6$$

| | | |
|---|-----|----|
| x | 30 | 7 |
| 6 | 180 | 42 |

$$\begin{array}{r} 180 \\ + 42 \\ \hline 222 \end{array}$$

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Y4

End of Year Objective:

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

Children will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number, e.g.

$$79 \times 8$$

| | | |
|---|-----|----|
| x | 70 | 9 |
| 8 | 560 | 72 |

$$\begin{array}{r} 560 \\ + 72 \\ \hline 632 \end{array}$$

By the end of the year, they will extend their use of the grid method to be able to multiply three-digit numbers by a single digit number, e.g.

346×8

| | | | |
|---|------|-----|----|
| x | 300 | 40 | 6 |
| 8 | 2400 | 320 | 48 |

$$\begin{array}{r}
 2400 \\
 + 320 \\
 + 48 \\
 \hline
 2768
 \end{array}$$

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Y5

End of Year Objective:

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

Children should continue to use the grid method and extend it to multiplying numbers with up to four digits by a single digit number, e.g.

4346×8

| | | | | |
|---|-----------|------|-----|----|
| x | 4
000 | 300 | 40 | 6 |
| 8 | 32
000 | 2400 | 320 | 48 |

$$\begin{array}{r}
 32000 \\
 + 2400 \\
 + 320 \\
 + 48 \\
 \hline
 34768
 \end{array}$$

and numbers with up to four digits by a two-digit number, e.g.

2693×24

| | | | | |
|----|-------|-------|------|----|
| x | 2000 | 600 | 90 | 3 |
| 20 | 40000 | 12000 | 1800 | 60 |
| 4 | 8000 | 2400 | 360 | 12 |

$$\begin{array}{r}
 40000 \\
 + 8000 \\
 + 12000 \\
 + 2400 \\
 + 1800 \\
 + 360 \\
 + 60 \\
 + 12 \\
 \hline
 64632
 \end{array}$$

The long list of numbers in the addition part can be used to check that all of the answers from the grid have been included, however, when children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they should be encouraged to do so.

For example,

| | | | | | |
|----|-------|-------|------|----|---------------|
| x | 2000 | 600 | 90 | 3 | |
| 20 | 40000 | 12000 | 1800 | 60 | = 53 860 |
| 4 | 8000 | 2400 | 360 | 12 | = 10 772 |
| | | | | | + |
| | | | | | <u>64 632</u> |

Adding across mentally, leads children to finding the separate answers to:

$$2\ 693 \times 20$$

$$2\ 693 \times 4$$

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

During Year 5, the transition from the grid method into the formal vertical method for multiplication could take place. The traditional vertical compact method of written multiplication is a highly efficient way to calculate, but it has a very condensed form and needs to be introduced carefully.

It is most effective to begin with the grid method, moving to an expanded vertical layout, before introducing the compact form. This allows children to see, and understand, how the processes relate to each other and where the individual multiplication answers come from e.g.

$$368 \times 6$$

| | | | | |
|---|-------|-----|----|--|
| x | 300 | 60 | 8 | |
| 6 | 1 800 | 360 | 48 | |

| | | |
|---|-------------|--|
| | 1800 | |
| + | 360 | |
| + | 48 | |
| | <u>2208</u> | |

| | | | |
|----|----------|----------|----------|
| Th | H | T | U |
| 3 | 6 | 8 | |
| x | | | 6 |
| | | 4 | 8 |
| | | 3 | 6 |
| + | 1 | 8 | 0 |
| | <u>2</u> | <u>2</u> | <u>0</u> |
| | | | 8 |

| | | | |
|----|----------|----------|----------|
| Th | H | T | U |
| 3 | 6 | 8 | |
| x | | | 6 |
| | | 4 | 8 |
| | | 3 | 6 |
| + | 1 | 8 | 0 |
| | <u>2</u> | <u>2</u> | <u>0</u> |
| | | | 8 |

| | | | | |
|---------|----|---|----------|----------|
| | Th | H | T | U |
| | 3 | 6 | 8 | |
| | x | | | 6 |
| becomes | | | <u>2</u> | <u>2</u> |
| | | | | 0 |
| | | | | 8 |

The place value columns are labelled to ensure children understand the size of the partitioned digits in the original number(s) and in the answer. It is vital that the teacher models the correct language when explaining the process of the compact method.

The example shown should be explained as:

"Starting with the least significant digit... 8 multiplied by 6 is 48, put 8 in the units and carry 4 tens (40).

6 tens multiplied by 6 are 36 tens. Add the 4 tens carried over to give 40 tens (which is the same as 4 hundreds and 0 tens). Put 0 in the tens place of the answer and carry 4 hundreds.

3 hundreds multiplied by 6 are 18 hundreds. Add the 4 hundreds carried over to give 22 hundreds (which is the same as 2 thousands and 2 hundreds). Write 2 in the hundreds place of the answer and 2 in the thousands place of the answer."

Children should recognise that the answer is close to an estimated answer of $400 \times 6 = 2400$

Long multiplication could also be introduced by comparing the grid method with the compact vertical method. Mentally totalling each row of answers is an important step in children making the link between the grid method and the compact method.

| | | | | |
|----|-------|------|----|---------------|
| x | 600 | 90 | 3 | |
| 20 | 12000 | 1800 | 60 | = 13 860 |
| 4 | 2400 | 360 | 12 | = 2 772 + |
| | | | | <u>16 632</u> |

Children should only be expected to move towards this next method if they have a secure understanding of place value. It

is difficult to explain the compact method without a **deep understanding of place value**.

Step 1

| | | | | | |
|---|----------|----------|----------|----------|---|
| | T | Th | H | T | U |
| | 6 | 9 | 3 | | |
| x | 2 | 4 | | | |
| | <u>2</u> | <u>7</u> | <u>7</u> | <u>2</u> | |

(693 x 4)

b ↓

The example shown should be explained as:

"Starting with the least significant digit... 3 multiplied by 4 is 12; put 2 in the units and carry 1 ten (10).

9 tens multiplied by 4 are 36 tens. Add the 1 ten carried over to give 37 tens (which is the same as 3 hundreds and 7 tens). Put 7 in the tens place of the answer and carry 3 hundreds.

6 hundreds multiplied by 4 are 24 hundreds. Add the 3 hundreds carried over to give 27 hundreds (which is the same as 2 thousands and 7 hundreds). Write 7 in the hundreds place of the answer and 2 in the thousands place of the answer. We have now found the answer to 693×4 . Step 1 is complete so to avoid confusion later, we will cross out the carried digits 3 and 1."

Notice this answer can clearly be seen in the grid method example.

Step 2

| | | | | |
|-----|----------|----------|----------|----------|
| TTh | Th | H | T | U |
| | 6 | 9 | 3 | |
| | <u>x</u> | <u>2</u> | <u>4</u> | |
| | 2 | 7 | 7 | 2 |
| + | 1 | 3 | 8 | 6 |
| | <u>0</u> | <u>0</u> | <u>0</u> | <u>0</u> |

(693 × 4)
(693 × 20)

Now we are multiplying 693 by 20. Starting with the least significant digit of the top number... 3 multiplied by 20 is 60. Write this answer in. 90 multiplied by 20 is 1 800. There are no units and no tens in this answer, so write 8 in the hundreds place and carry 1 in the thousands. 600 multiplied by 20 is 12 000. Add the 1 (thousand) that was carried to give 13 000. There are no units, no tens and no hundreds in this answer, so write 3 in the thousands place and 1 in the ten thousands place.

Step 3

| | | | | |
|-----|----------|----------|----------|----------|
| TTh | Th | H | T | U |
| | 6 | 9 | 3 | |
| | <u>x</u> | <u>2</u> | <u>4</u> | |
| | 2 | 7 | 7 | 2 |
| + | 1 | 3 | 8 | 6 |
| | <u>0</u> | <u>0</u> | <u>0</u> | <u>0</u> |
| | 1 | 6 | 6 | 3 |
| | | | | 2 |

(693 × 4)
(693 × 20)

The final step is to total both answers using efficient columnar addition.

When using the compact method for long multiplication, all carried digits should be placed below the line of that answer e.g. 3 × 4 is 12, so the 2 is written in the units column and the 10 is carried as a small 1 in the tens column. This carrying below the answer is in line with the written addition policy in which carried digits are always written below the answer/line.

Y6

End of Year Objective:

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.

Multiply one-digit numbers with up to two decimal places by whole numbers.

By the end of year 6, children should be able to use the grid method to multiply any number by a two-digit number. They could also develop the method to be able to multiply decimal numbers with up to two decimal places, but having been introduced to expanded and compact vertical methods in Year 5, it may be appropriate to use the expanded vertical method when introducing multiplication involving decimals.

$$4.92 \times 3$$

$$\begin{array}{r} \text{T U . t h} \\ 4 . 9 2 \\ \times \quad 3 \\ \hline 0 . 0 6 \quad (0.02 \times 3) \\ 2 . 7 \quad (0.9 \times 3) \\ + 1 2 \quad (4 \times 3) \\ \hline \underline{14.76} \end{array}$$

becomes

$$\begin{array}{r} \text{T U . t h} \\ 4 . 9 2 \\ \times \quad 3 \\ \hline \underline{14.76} \\ 2 \end{array}$$

Children should also be using this method to solve problems and multiply numbers, including those with decimals, in the context of money or measures, e.g. to calculate the cost of 7 items at £8.63 each, or the total length of six pieces of ribbon of 2.28m each.

Progression Towards a Written Method for Division

In developing a written method for division, it is important that children understand the concept of division, in that it is:

- repeated subtraction

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of multiplication
- not commutative i.e. $15 \div 3$ is not the same as $3 \div 15$
- not associative i.e. $30 \div (5 \div 2)$ is not the same as $(30 \div 5) \div 2$

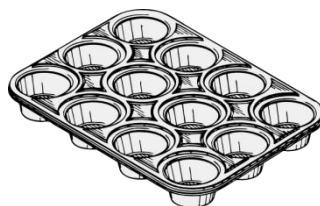
YR

Early Learning Goal:

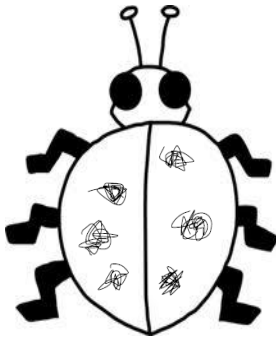
Children solve problems, including halving and sharing.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.



A child's jotting showing halving six spots between two sides of a ladybird.



A child's jotting showing how they shared the apples at snack time between two groups.

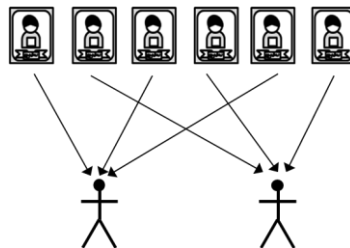


Y1

End of Year Objective:

Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

In year one, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two people, how many do they each get?' They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.



Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '... left over'.

Y2

End of Year Objective:

Calculate mathematical statements for division within the multiplication tables and write them using the division (\div) and equals (=) signs.

Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

$12 \div 3 =$

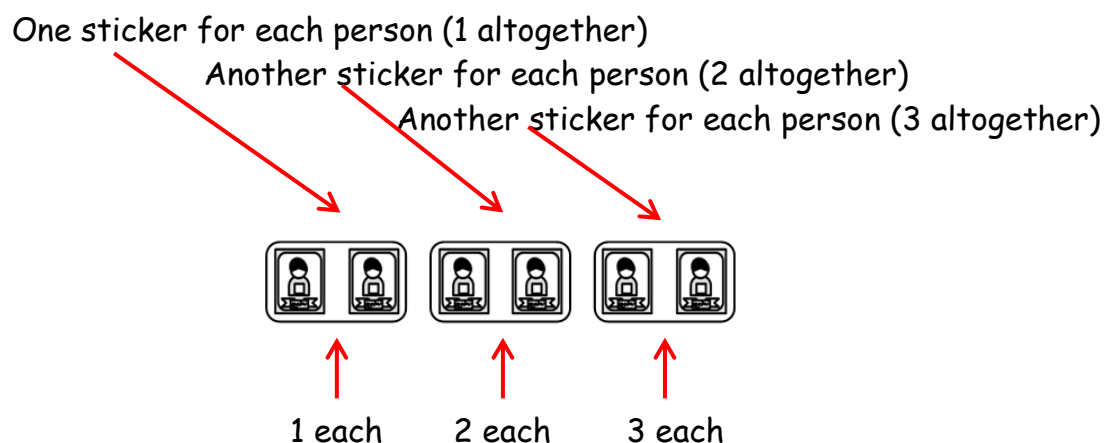


Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'

The link between sharing and grouping can be modelled in the following way:

To solve the problem 'If six football stickers are shared between two people, how many do they each get?'

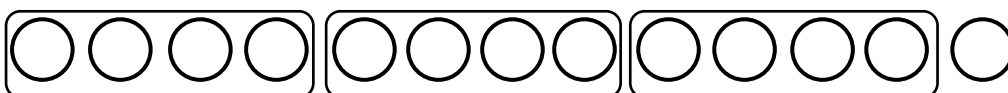
Place the football stickers in a bag or box and ask the children how many stickers would need to be taken out of the box to give each person one sticker each (i.e. 2) and exemplify this by putting the cards in groups of 2 until all cards have been removed from the bag.



Or:

Children should also continue to develop their knowledge of division with remainders, e.g.

$13 \div 4 =$



$13 \div 4 = 3 \text{ remainder } 1$

Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation $13 \div 4$, the answer is 3 remainder 1,

but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:

I have £13. Books are £4 each. How many can I buy?

Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed?

Answer: 4 (the remaining 1 apple still need to be placed into a box)

Y3

End of Year Objective:

Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods.*

Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$$43 \div 8 =$$



$$43 \div 8 = 5 \text{ remainder } 3$$

In preparation for developing the 'chunking' method of division, children should first use the repeated subtraction on a vertical number line alongside the continued use of practical equipment. There are two stages to this:

Stage 1 - repeatedly subtracting individual groups of the divisor

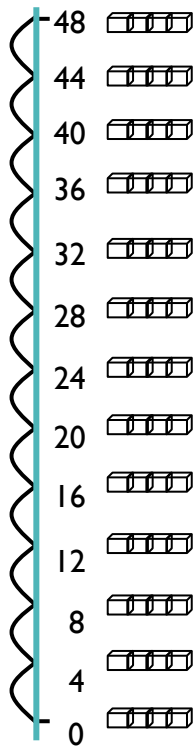
Stage 2 - subtracting multiples of the divisor (initially 10 groups and individual groups, then 10 groups

and other multiples in line with tables knowledge)

After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line.

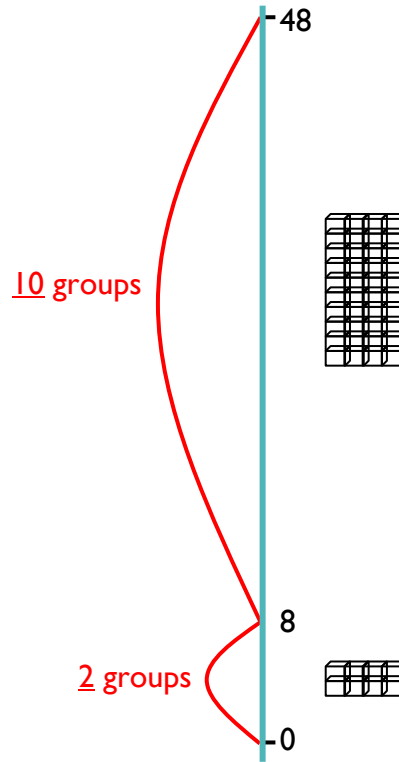
Stage 1

$$48 \div 4 = 12 \text{ (groups of 4)}$$



Stage 2

$$48 \div 4 = 10 \text{ (groups of 4)} + 2 \text{ (groups of 4)} \\ = 12 \text{ (groups of 4)}$$



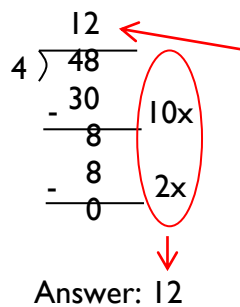
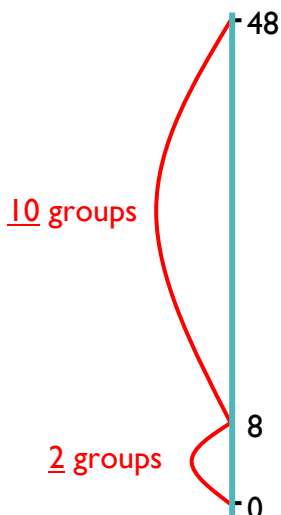
Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Y4

End of Year Objective:

Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Children will continue to develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, moving on to the use of the 'chunking' method.



Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

The number line method used in year 3 can be linked to the chunking method to enable children to make links in their understanding. Page 23 of 26

When developing their understanding of 'chunking', children should utilise a 'key facts' box, as shown below. This enables an efficient recall of tables facts and will help them in identifying the largest group they can subtract in one chunk. Any remainders should be shown as integers, e.g.

$$73 \div 3$$

$$\begin{array}{r}
 24r1 \\
 3 \overline{) 73} \\
 - 30 \\
 \hline
 43 \\
 - 30 \\
 \hline
 13 \\
 - 6 \\
 \hline
 7 \\
 - 6 \\
 \hline
 1
 \end{array}$$

Key facts box

| | |
|-----|----|
| 1x | 3 |
| 2x | 6 |
| 5x | 15 |
| 10x | 30 |

By the end of year 4, children should be able to use the chunking method to divide a three digit number by a single digit number. To make this method more efficient, the key facts in the menu box should be extended to include 4x and 20x, e.g.

$$196 \div 6$$

$$\begin{array}{r}
 32r4 \\
 6 \overline{) 196} \\
 - 120 \\
 \hline
 76 \\
 - 60 \\
 \hline
 16 \\
 - 12 \\
 \hline
 4
 \end{array}$$

Key facts box

| | |
|-----|-----|
| 1x | 6 |
| 2x | 12 |
| 4x | 24 |
| 5x | 30 |
| 10x | 60 |
| 20x | 120 |

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Y5

End of Year Objective:

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Using their knowledge of linked tables facts, children should be encouraged to use higher multiples of the divisor. During Year 5, children should be encouraged to be efficient when using the chunking method and not have any subtraction steps that repeat a previous step. For example, when performing $347 \div 8$ an initial subtraction of 160 (20×8) and a further subtraction of 160 (20×8) should be changed to a single subtraction of 320 (40×8). Children will continue to use the chunking method to divide by a one digit number early in year 5, working towards using this method to divide by a 2 digit number.

By the end of year 5, children will have moved on to using the more efficient method of division when dividing a number by a one digit number.

$$\begin{array}{r} \underline{351} \text{ r } 1 \\ 7 \overline{) 2458} \end{array}$$

$$2400 \div 7 = 300 \text{ r } 30$$

$$350 \div 7 = 50$$

$$8 \div 7 = 1 \text{ r } 1$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Y6

End of Year Objective:

Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

Use written division methods in cases where the answer has up to two decimal

To develop the chunking method further, it should be extended to include dividing a four-digit number by a two-digit number, e.g.

$$6367 \div 28$$

$$\begin{array}{r} \underline{227} \text{ r } 11 \\ 28 \overline{) 6367} \\ - \underline{5600} \\ \quad 767 \\ - \underline{560} \\ \quad \quad 207 \\ - \underline{140} \\ \quad \quad \quad 67 \\ - \underline{56} \\ \quad \quad \quad \quad 11 \end{array}$$

200x
20x
5x
2x

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

In addition, children should also be able to use the chunking method and solve calculations interpreting the remainder as a decimal up to two decimal places.

This should first be demonstrated using a simple calculation such as $13 \div 4$ to show the remainder initially as a fraction.



Using practical equipment, children can see that for $13 \div 4$, the answer is 3 remainder 1, or put another way, there are three whole groups and a remainder of 1. This remainder is one part towards a full group of 4, so is $\frac{1}{4}$. To show the remainder as a fraction, it becomes the numerator where the denominator is the divisor (the number that you are dividing by in the calculation).

$$3574 \div 8$$

$$\begin{array}{r} 8 \overline{) 3574} \\ - 3200 \quad 400x \\ \hline 574 \\ - 560 \quad 70x \\ \hline 14 \\ - 8 \quad 1x \\ \hline 6 \end{array}$$

$$\begin{array}{r} 6 \leftarrow \text{remainder} \\ - 8 \leftarrow \text{divisor} \end{array}$$

So $3574 \div 8$ is $471\frac{6}{8}$
(when the remainder is shown as a fraction)

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half: $\frac{1}{2} = 0.5$

Quarters: $\frac{1}{4} = 0.25$, $\frac{3}{4} = 0.75$

Fifths: $\frac{1}{5} = 0.2$, $\frac{2}{5} = 0.4$, $\frac{3}{5} = 0.6$, $\frac{4}{5} = 0.8$

Tenths: $\frac{1}{10} = 0.1$, $\frac{2}{10} = 0.2$, $\frac{3}{10} = 0.3$, $\frac{4}{10} = 0.4$, $\frac{5}{10} = 0.5$, $\frac{6}{10} = 0.6$, $\frac{7}{10} = 0.7$, $\frac{8}{10} = 0.8$, $\frac{9}{10} = 0.9$

and reduce other equivalent fractions to their lowest terms.

In the example above, $3574 \div 8$, children should be able to identify that the remainder as a fraction of $\frac{6}{8}$ can be written as $\frac{3}{4}$ in its lowest terms. As $\frac{3}{4}$ is equivalent to 0.75, the answer can therefore be written as 471.75.